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A single-equation solution for conduction in fins

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Abstract—A single-equation solution is presented for solving temperature and heat flux in many finite one-dimensional bodies with fin effect. When the fin effect is set equal to zero, it yields the temperature and heat flux in finite one-dimensional bodies. The bodies can be single layer or multilayer. The single-equation solution is for transient conduction but yields the steady-state solution at large times. A recently developed alternative Green's function solution method is used. The Green's function is calculated using the Galerkin-based integral method. The examples in this paper are for one- and two-layer bodies.

INTRODUCTION

A RECENT paper [1] introduced a technique for solving a variety of linear multidimensional thermal conduction problems by a single equation. The single equation solution in this paper modifies the Alternative Green's Function Solution (AGFS) method [1, 2] to include the fin effect. The AGFS uses an auxiliary source term in the solution equation. The Galerkin-based Integral (GBI) method [3] yields the Green's function relation in the AGFS [1]. It is shown that minor changes will extend AGFS [1] to include the fin effect. Four thermal conduction examples for one-dimensional bodies with fin effect are presented. The method can include multidimensional bodies but this paper only addresses one-dimensional bodies. The product method provides the Green's functions in some regular multidimensional bodies.

The single-equation solution method applies to the diffusion equation in various finite bodies and in various coordinate systems. The procedure is described and the accuracy is compared with the exact solution for a selected case. The observed accuracy is excellent and exceeds expectations. The single-equation solution yields accurate temperature and heat flux on a personal computer immediately after the data are entered.

The single-equation approach is not limited to fins; it can yield temperature and heat flux in solid cylinders and spheres for the diffusion equation in bioengineering applications. Ignoring the fin effect results in the solution of standard one-dimensional conduction problems. The solids under consideration can be multilayered but only one example considers a two-layer body. The solution holds for transient and steady-state problems.

ANALYSIS

The transient diffusion equation for one-dimensional problems used in this paper is

$$\nabla\cdot[k(r)\nabla T]+g(r,t)-w(r)T = \rho(r)c_p(r)\partial T/\partial t \qquad (1)$$

where $T = T(r, t)$ is the temperature, g the volumetric heat source, ρ the density, c_p the specific heat, k the thermal conductivity, and r represents Cartesian, cylindrical, or spherical coordinates. The term $\nabla\cdot[k(r)\nabla T]$ on the left-hand side of equation (1), in this paper, is $(1/r^p)\partial[k(r)r^p\partial T/\partial r]/\partial r$, where $p = 0$, 1, or 2 for Cartesian, cylindrical, or spherical coordinates. The coefficient p takes other positive real numbers for fins. The thermophysical properties $\rho(r)$, $c_p(r)$, and $k(r)$ are position-dependent density, specific heat, and thermal conductivity. The term $w(r)T$ is the fin convective effect.

The alternative Green's function solution equation for heterogeneous materials [1] is

$$T(r,t) = T^*(r,t)+\frac{1}{\rho(r)c_p(r)} \times\left\{\int_{\tau=0}^{t}\mathrm{d}\tau\int_V G[f^*+g(r',\tau) -\rho(r')c_p(r')\partial T^*(r',\tau)/\partial\tau]\,\mathrm{d}V' +\int_V \rho(r')c_p(r')G|_{\tau=0}[F(r')-T^*(r',0)]\,\mathrm{d}V'\right\} \qquad (2)$$

where $G = G(r', -\tau|r, -t) = G(r', t|r, \tau)$ is given by

$$G(r',t|r,\tau) = \rho(r)c_p(r)\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{i=1}^{N} d_{nj}p_{ni} \times\exp[-\gamma_n(t-\tau)]f_j(r')f_i(r). \qquad (3)$$

The parameters p_{ni} and d_{nj} are documented in ref. [3]. The fin effect does not change equation (2) or (3) and the relations given in ref. [1] remain unchanged.

The function T^* is an auxiliary function defined to satisfy the non-homogeneous boundary conditions but it is not necessarily the steady-state or quasi-steady-state solution. Only the function T^* contains

NOMENCLATURE

a, b geometrical dimensions [m]
a_{ij} element of matrix $\mathbf{A}$, equation (11)
$\mathbf{A}$ matrix
b_{ij} element of matrix $\mathbf{B}$, equation (12)
$\mathbf{B}$ matrix
B_1, B_2 ratio of $h_1/k_1, h_2/k_2$
c_p specific heat [J kg^{-1} K^{-1}]
C conductance [W m^{-2} K^{-1}]
d_{nj} coefficients, equation (3)
$\mathbf{d}_n$ eigenvector with elements d_{nj}, equation (10)
$\mathbf{D}$ matrix with element d_{nj}
f_j basis functions
f^* function defined in equation (4)
$F(r)$ initial temperature distribution
g energy generation per unit time and per unit volume [W m^{-3}]
G Green's function, $G(r', -\tau|r, -t)$
h heat transfer coefficient [W m^{-2} K^{-1}]
i, j indices
k thermal conductivity [W m^{-1} K^{-1}]
L length [m]
m factor, $(w/k)^{0.5}$
n index
N number of eigenvalues
p index
p_{ni} element of matrix $\mathbf{P}$
$\mathbf{P}$ inverse of transpose of matrix $(\mathbf{DB})$
r radial coordinate
r' dummy variable
S surface
t time [s]
T temperature [K]
T_i initial temperature distribution [K]
T_s surface temperature [K]
T^* auxiliary solution, e.g. equation (5)
V volume [m^3]
w fin factor [W m^{-3} K^{-1}]
x Cartesian coordinate [m].

Greek symbols
α thermal diffusivity [m^2 s^{-1}]
γ_n eigenvalues
ρ density [kg m^{-3}]
τ time; also dummy variable.

Subscripts
1 parameter at small dimension
2 parameter at large dimension
b temperature of the fin at the base
o fluid temperature.

the contribution of non-homogeneous boundary conditions. The function f^*, appearing as a source term in equation (2), compensates for the arbitrary nature of T^* and is given by the relation

$$f^*(r', \tau) = \nabla_o \cdot [k\nabla_o T^*(r', \tau)] - w(r)T^* \tag{4}$$

where ∇_o implies the derivatives are in r' space. If $f^*(r', \tau) = 0$ and $w(r) = 0$, then $T^*(r', \tau)$ satisfies the Laplace equation and it is the quasi-steady solution. Note that the function $f^*(r', \tau)$ defined by equation (4) is unrelated to the basis functions $f_j(r)$ in equation (3). The role of the function f^* is to improve the convergence of the Green's function solution.

The procedure for defining f^* is discussed in refs. [1, 2]. The procedure begins by defining a differentiable temperature function that satisfies the non-homogeneous boundary conditions of the first, second, and third kinds. The function T^* is

$$T^* = c_1 u_p(r) + c_2. \tag{5}$$

In one-dimensional coordinates, the function u_p takes the value of x in Cartesian coordinates (r becomes x), $\ln(r)$ in radial cylindrical coordinates and $-1/r$ in spherical coordinates. For a prescribed heat flux at both surfaces, the term $c_2 r^2$ replaces c_2. The calculation of c_1 and c_2 for non-homogeneous boundary conditions is elementary and requires solving two equations for two unknowns, c_1 and c_2.

COMPUTATIONAL STEPS

The steps leading to the temperature and heat flux solution in sequential order are given below.

Basis functions

The computation begins by defining a set of basis functions that satisfies the homogeneous boundary conditions

$$k_1\,\mathrm{d}f_j/\mathrm{d}r = h_1 f_j \quad \text{at} \quad r = a \tag{6a}$$

$$-k_2\,\mathrm{d}f_j/\mathrm{d}r = h_2 f_j \quad \text{at} \quad r = b \tag{6b}$$

as

$$f_j = (\delta_j r^2 + \beta_j r + \eta_j) r^{j-1}; \quad \text{for } j = 1, 2, \ldots, N. \tag{7}$$

In Cartesian coordinates, r is replaced by x.

The boundary conditions, equations (6a) and (6b), determine the three coefficients, δ_j, β_j, and η_j. The δ_j coefficient can be selected arbitrarily. The δ_j coefficient is set equal to the determinant of the coefficients in the two equations when solving for β_j and η_j, resulting in

$$\delta_j = a(j - aB_1)(j - 1 + bB_2) - b(j + bB_2)(j - 1 - aB_1) \tag{8a}$$

$$\beta_j = a^2(aB_1 - j - 1)(j - 1 + bB_2) + b^2(bB_2 + j + 1)(j - 1 - aB_1) \tag{8b}$$

and

$$\eta_j = -ab^2(j-aB_1)(bB_2+j+1)-ba^2(j+bB_2) \times (aB_1-j-1) \quad \text{for } j = 1,2,3,\ldots,N \quad (8c)$$

where the parameters, B_1 and B_2, in equations (8a)–(8c) are h_1/k_1 and h_2/k_2. In special cases, when the $r = a$ surface is insulated, $B_1 = 0$; when the $r = b$ surface is insulated, $B_2 = 0$. If both surfaces, $r = a$ and b, are insulated, then $B_1 = B_2 = 0$. When the boundary conditions are of the first kind, the respective heat transfer coefficients become infinite. There are several ways of removing singularities: one is using the L'Hospital rule; another is to recalculate the parameters δ_j, β_j, and η_j using boundary condition(s) of the first kind. For instance, the coefficients in equation (8) when $B_1 = \infty$ and $B_2 = \infty$ are

$$\delta_j = 1, \quad \beta_j = -(a+b), \quad \text{and } \eta_j = ab, \quad \text{for } j = 1,2,3,\ldots,N. \quad (9)$$

Equations (7) and (8) hold for all one-dimensional conduction problems with homogeneous boundary conditions in finite domains.

The polynomials are used to construct the basis functions because they simplify all integrations leading to computation of the eigenvalues. The degree of polynomials should not exceed 10, otherwise matrices may become ill-conditioned. Polynomials of degree 5–7 usually yield accurate results for most practical applications. Other polynomials such as Legendre and Chebyshev can be used but their contribution is not cost effective when the degree of the polynomial is small.

Eigenvalues and eigenvectors

The computation of the eigenvalues and eigenvectors [1, 3] must be modified to include the fin effect contribution. Although the equation that yields the eigenvalues

$$(\mathbf{A}+\gamma_n\mathbf{B})\mathbf{d}_n = \mathbf{0} \quad (10)$$

remains unchanged, the elements of matrix **A** are modified by the fin effect

$$a_{ij} = \int_V f_i\nabla\cdot(k\nabla f_j)\,\mathrm{d}V - \int_V w(r)f_if_j\,\mathrm{d}V. \quad (11)$$

The elements of matrix **B**

$$b_{ij} = \int_V \rho(r)c_p(r)f_if_j\,\mathrm{d}V \quad (12)$$

will be unaffected by the fin effect. When k, ρ, c_p, and w are constant in any material layer, the identities presented in ref. [1] provide the values of integrals. The same identities can be used if these properties are described by polynomials. The coefficients d_{n1}, $d_{n2}, \ldots, d_{nN}$ in equation (10) are the elements of the eigenvector $\mathbf{d}_n$. The eigenvalues and eigenvectors are computed using the Jacobi method [4]. The computation details are given in ref. [5]. The second term on the right-hand side of equation (11) vanishes in the absence of the fin effect. Accordingly, the fin effect influences only matrix **A** and function f^*.

The coefficients d_{nj} and p_m in equation (3) are the elements of matrices **D** and **P**. The rows of matrix **D** are the eigenvectors computed from equation (10). The transpose of matrix **DB**, after it is inverted, becomes matrix **P**.

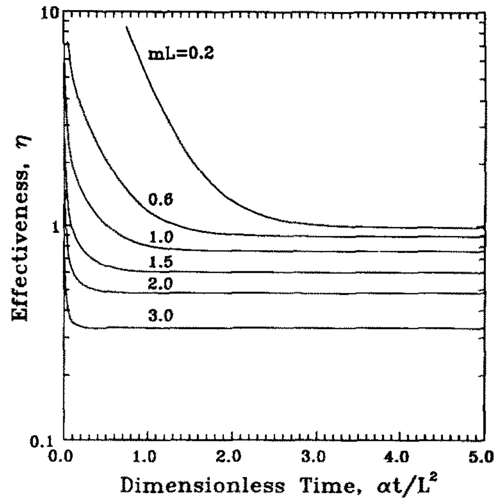

FIG. 1. Effectiveness, η, of straight fins with constant cross-sectional area as a function of dimensionless time, $\alpha t/L^2$.

RESULTS

The accuracy of the numerical results is illustrated in Examples 1 and 2. Examples 3 and 4 show the scope of this single-equation solution method.

Example 1

A simple example with a well-known solution is selected to verify the accuracy of the results. A straight fin with constant cross section and length L has a simple steady-state solution. When $T = 1$ at $x = 0$, and $q = 0$ at $x = L$ (insulated tip), the steady-state temperature is $\cosh[m(L-x)]/\cosh(mL)$ and the effectiveness is $\eta = \tanh(mL)/mL$, where $m = (hP/kA)^{1/2}$, P is the perimeter, A the cross-sectional area, h the heat transfer coefficient, and k the thermal conductivity. The value of w in equation (1) is hP/A. Equation (2) is used to calculate the transient and steady-state temperature field. Heat flux, and subsequently the fin effectiveness, are calculated as a function of $\alpha t/L^2$. The results are shown in Fig. 1. The data show that fins assume steady-state operation faster as the value of m becomes larger.

The asymptotic values of effectiveness, as time becomes infinite, are given in Table 1 for comparison with the exact solution. The values $c_1 = 0$ and $c_2 = T_b = 1$ are used for calculating T^* in equation (5). For all values of m, the data compare surprisingly well despite using a constant value for T^*. Table 1 indicates that for five eigenvalues ($N = 5$), results

Table 1. Effectiveness using GBI method with different N values for straight fins with uniform area

mL	GBI solution $N = 2$	$N = 5$	$N = 7$	Exact solution
0.0	1.00000	1.00000	1.00000	1.00000
0.2	0.98645	0.98688	0.98688	0.98688
0.4	0.94819	0.94988	0.94988	0.94987
0.6	0.89150	0.89508	0.89508	0.89508
0.8	0.82413	0.83005	0.83005	0.83005
1.0	0.75312	0.76159	0.76159	0.76159
1.2	0.68358	0.69472	0.69472	0.69471
1.4	0.61862	0.63240	0.63420	0.63239
1.6	0.55969	0.57605	0.57605	0.57604
1.8	0.50719	0.52599	0.52602	0.52600
2.0	0.46083	0.48200	0.48203	0.48201
2.2	0.42008	0.44349	0.44351	0.44352
2.4	0.38427	0.40983	0.40986	0.40986
2.6	0.35278	0.38034	0.38040	0.38040
2.8	0.32497	0.35443	0.35452	0.35451
3.0	0.30033	0.33158	0.33169	0.33169
3.5	0.24977	0.28500	0.28519	0.28519
4.0	0.21098	0.24951	0.24983	0.24983
4.5	0.18049	0.22169	0.22216	0.22217
5.0	0.15605	0.19931	0.19997	0.19998
6.0	0.11965	0.16550	0.16662	0.16667
8.0	0.07597	0.12250	0.12484	0.12500
10.0	0.05198	0.09591	0.09961	0.10000

obtained using the single-equation solution are as accurate as those achieved by the exact solution. Notice that $T^* = 1$ is a function that only satisfies the boundary conditions; it is not the steady-state solution.

Example 2

To show the utility of this single-equation solution, consider a cylindrical body, $p = 1$, with fin effect. Here, the boundary conditions are: $T = T_h = 1$ at $r = r_1$ and $q = 0$ at $r = r_2$. It is indifferent to this solution whether the heat flux or temperature is given at the boundary. For boundary conditions of the third kind at one or both boundaries, no additional steps are required. Similar to Example 1, equation (1) is used to obtain this solution. Except for the input data, no other changes are made. Figure 2 shows the effectiveness as a function of dimensionless time, $\alpha t/(r_2-r_1)^2$, for different values of $m = r_1(w/k)^{0.5}$.

To show the accuracy obtainable with the single-equation solution, the steady-state effectiveness for different r_2/r_1 ratios, for a range of values of m are shown in Table 2. The data compare well with the exact solution; usually up to five significant figures. All entries in Table 2 are for $N = 9$. Similar to Table 1, Table 3 contains the effectiveness calculated for different values of N. Only one value of r_2/r_1 is used in this presentation. Tables 1 and 3 show that, when $N = 5$, sufficient accuracy is achieved for nearly all practical applications.

The number of problems that can be solved using this single-equation solution method is so large, only a few examples can be presented. It accommodates Cartesian, cylindrical, and spherical coordinates.

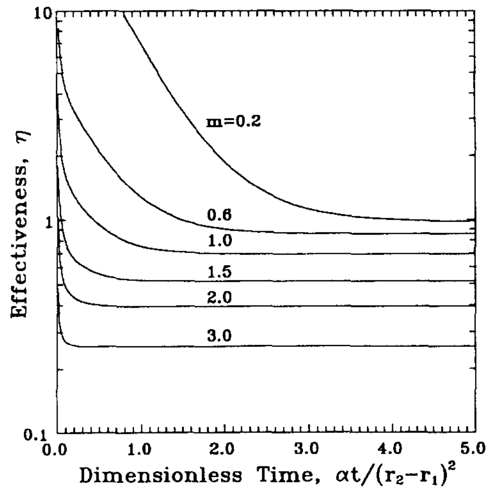

FIG. 2. Effectiveness, η, of circular fins with rectangular profile as a function of dimensionless time, $\alpha t/(r_2-r_1)^2$; $r_2/r_1 = 2$.

Example 3 describes heat conduction in a non-elementary fin. The fin effect in cylindrical and spherical bodies is encountered in some heat transfer applications in bioengineering. Example 4 presents a case where the fin effect exists in a body without fins.

Example 3

This example demonstrates the broad range of problems that submit to this single-equation solution. A circular pin fin the radius, r, of which varies as x^2, when $0.5 < x/x_2 < 1$ is considered. The perimeter varies as x^2 and the heat transfer coefficient as $x^{-0.5}$. The boundary conditions are convective with $hx_2/k = 0.02$ at $x = x_1 = x_2/2$ and $(T-T_i)/(T_h-T_i) = 1$ at the base. If the heat transfer coefficient at the fin's surface varies as $r^{-0.25}$ or $x^{-0.5}$, then the quantity hP varies as $x^{1.5}$. Accordingly, in this example, it is assumed that $x_2^2(hP/kA) = 2x^{-2.5}$; the factor 2 is arbitrary. The temperature at various locations along the fin is plotted as a function of $\alpha t/x_2^2$ in Fig. 3. The same single-equation solution, with no modifications, that produced the data presented in Examples 1 and 2 was used to provide the data presented in Fig. 3. The accuracy of these data is expected to be comparable with those for the previous examples.

Example 4

The more general form of the basis functions is given in ref. [1]. The generalized form of the basis functions permits solution for layered materials with perfect or imperfect contact between layers. Consider a two-layer cylinder the inner and outer radii of which are r_1 and r_2; $r_1/r_2 = 0.6$. The thicknesses of the layers are equal. The inner layer is porous and the outer layer is impermeable. It is assumed that the heat transfers from the porous material to the working fluid. The other input parameters for this problem are selected mainly to demonstrate the range of capability

Table 2. Fin effectiveness using GBI method and comparison with the exact solution for cylindrical fins

m	$r_2/r_1 = 1.2$ GBI	$r_2/r_1 = 1.2$ Exact	$r_2/r_1 = 1.5$ GBI	$r_2/r_1 = 1.5$ Exact	$r_2/r_1 = 2.0$ GBI	$r_2/r_1 = 2.0$ Exact	$r_2/r_1 = 3.0$ GBI	$r_2/r_1 = 3.0$ Exact
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.2	0.99941	0.99942	0.99592	0.99593	0.98148	0.98146	0.91601	0.91607
0.4	0.99766	0.99767	0.98392	0.98393	0.93025	0.93024	0.73696	0.73700
0.6	0.99477	0.99477	0.96466	0.96467	0.85710	0.85713	0.56485	0.56487
0.8	0.99074	0.99075	0.93913	0.93914	0.77434	0.77434	0.43493	0.43494
1.0	0.98563	0.98563	0.90852	0.90853	0.69153	0.69154	0.34349	0.34351
1.2	0.97946	0.97946	0.87413	0.87414	0.61464	0.61464	0.27932	0.27934
1.4	0.97229	0.97229	0.83723	0.83724	0.54630	0.54629	0.23330	0.23332
1.6	0.96416	0.96417	0.79900	0.79900	0.48705	0.48706	0.19933	0.19935
1.8	0.95515	0.95515	0.76035	0.76035	0.43642	0.43641	0.17351	0.17354
2.0	0.94531	0.94531	0.72211	0.72213	0.39335	0.39332	0.15338	0.15340
2.2	0.93471	0.93471	0.68493	0.68493	0.35667	0.35667	0.13728	0.13731
2.4	0.92341	0.92342	0.64915	0.64917	0.32543	0.32542	0.12415	0.12419
2.6	0.91151	0.91152	0.61513	0.61515	0.29864	0.29863	0.11328	0.11331
2.8	0.89905	0.89906	0.58300	0.58302	0.27557	0.27555	0.10410	0.10414
3.0	0.88612	0.88613	0.55282	0.55286	0.25553	0.25553	0.09629	0.09633
3.5	0.85215	0.85217	0.48590	0.48595	0.21573	0.21573	0.08099	0.08105
4.0	0.81665	0.81667	0.43018	0.43024	0.18632	0.18631	0.06980	0.06991
4.5	0.78049	0.78053	0.38398	0.38406	0.16378	0.16381	0.06133	0.06144
5.0	0.74443	0.74448	0.34559	0.34569	0.14607	0.14609	0.05460	0.05479
6.0	0.67474	0.67482	0.28642	0.28660	0.12000	0.12003	0.04471	0.04501
8.0	0.55316	0.55334	0.21159	0.21200	0.08821	0.08840	0.03249	0.03315
10.0	0.45861	0.45895	0.16783	0.16780	0.06953	0.06992	0.02510	0.02622

Table 3. Effectiveness using GBI method with different N values for straight cylindrical fins, $r_2/r_1 = 2$

m	GBI solution $N = 2$	GBI solution $N = 5$	GBI solution $N = 7$	Exact solution
0.0	1.00000	1.00000	1.00000	1.00000
0.2	0.95567	0.98130	0.98145	0.98146
0.4	0.90400	0.93008	0.93021	0.93024
0.6	0.83024	0.85698	0.85713	0.85713
0.8	0.74667	0.77418	0.77432	0.77434
1.0	0.66302	0.69140	0.69153	0.69154
1.2	0.58523	0.61449	0.61463	0.61464
1.4	0.51605	0.54612	0.54629	0.54629
1.6	0.45599	0.48688	0.48706	0.48706
1.8	0.40453	0.43621	0.43640	0.43641
2.0	0.36067	0.39312	0.39332	0.39332
2.2	0.32328	0.35645	0.35667	0.35667
2.4	0.29133	0.32516	0.32541	0.32542
2.6	0.26388	0.29835	0.29862	0.29863
2.8	0.24018	0.27525	0.27553	0.27555
3.0	0.21958	0.25519	0.25552	0.25553
3.5	0.17851	0.21528	0.21571	0.21573
4.0	0.14811	0.18572	0.18628	0.18631
4.5	0.12490	0.16305	0.16378	0.16381
5.0	0.10673	0.14515	0.14603	0.14609
6.0	0.08042	0.11864	0.11991	0.12003
8.0	0.04999	0.08590	0.08807	0.08840
10.0	0.03381	0.06622	0.06923	0.06992

of this single-equation solution. Without any reference to specific applications, the parameters are $k_1/k_2 = 0.5$, $(\rho c_p)_1/(\rho c_p)_2 = 1$, $w_1 r_2^2/k_2 = 0.05$, $w_2 r_2^2/k_2 = 0$, $g_1 r_2^2/k_2 = 0.1$, and $g_2 r_2^2/k_2 = 0$. The dimensionless heat transfer coefficients at $r = r_1$ and $r = r_2$ are $h_1 r_2/k_2 = 2$ and $h_2 r_2/k_2 = 5$. The contact between the two layers is imperfect and the dimensionless contact conductance is $Cr_2/k_2 = 0.05$. The fluid temperature inside the inner surface of the cylinder is T_{o1}.

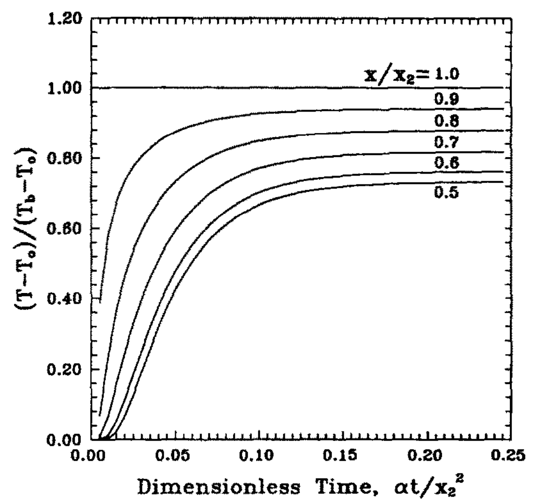

FIG. 3. Temperature at various locations along a pin fin as a function of dimensionless time, $\alpha t/x_2^2$.

The heat loss by fin effect is also to the same inside fluid. The temperature of fluid external to this cylinder is T_{o2} and the initial temperature is T_i. In the dimensionless form, the initial temperature $(T_i - T_{o1})/(T_{o2} - T_{o1}) = -1$ is considered. Figure 4 shows the temperature distribution, $(T - T_{o1})/(T_{o2} - T_{o1})$, as a function of position, r/r_2, for different values of $\alpha t/r_2^2$. The computer program used for the previous examples yields each curve in Fig. 4 within 1 s on a personal computer system with Intel 80386-20/387 processors. The data in Fig. 4 are accurate for $\alpha t/r_2^2 \geqslant 0.001$. When $\alpha t/r_2^2$ is much smaller, an approximate small time solution can be used.

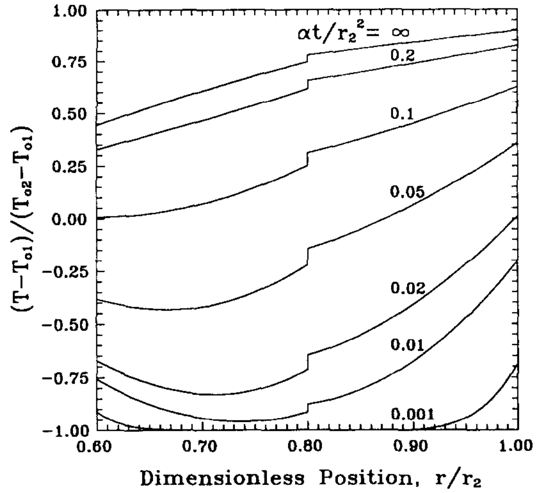

FIG. 4. Temperature distribution in a two layer, hollow cylinder as a function of position, r/r_2, for different values of $\alpha t/r_2^2$.

CONCLUSION AND REMARKS

The solution method presented in ref. [1], after a minor modification, leads to a single equation for solving most linear one-dimensional conduction problems. The alternative Green's function solution method is extended to include the contribution of the fin effect. Many different combinations of boundary conditions and configurations can be solved by this single-equation solution method. This is a new and unique tool for research and teaching of heat transfer. A personal computer and the computer program developed for the single-equation technique yields solutions to a multitude of problems. The exact solution for many of these problems is formidable. The speed of computing difficult problems is even faster than the exact solutions the eigenvalue equation of which is transcendental. The computation time for the basis functions, eigenvalues, and matrices for the Green's function depends on the number of basis functions, e.g. for 2, 5, and 7 basis functions, the computation times are 0.1, 0.8, and 1.7 s. The accuracy, as shown in Examples 1 and 2, is more than sufficient for most practical applications.

Despite the arbitrary selection of the function T^* and subsequently f^*, the solution given by equation (2) is exact if the Green's functions are exact. The approximate single Green's function relation produced by the Galerkin-based integral method results in a single equation that yields accurate solutions for most linear one-dimensional conduction problems. Despite the approximate nature of this single Green's function relation, the accuracy of the heat-flux data shown in Tables 1 and 2 is surprisingly good and reveals the unique potential of this single-equation solution approach.

The single-equation solution method is derived for linear one-dimensional conduction problems. There are some two-dimensional problems that can be solved by the product solution of one-dimensional problems. A few non-linear problems can be accommodated indirectly; however, these are not essential to this presentation and are not addressed here.

Further information on the Galerkin method can be found in Kantorovich and Krylov [6]. The use of the Galerkin method to solve the diffusion equation was recognized decades ago [7]. It never received serious attention. The Green's function using the Galerkin-based integral method was first reported in ref. [8].

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SOLUTION D'UNE EQUATION UNIQUE POUR LA CONDUCTION DANS LES AILETTES

Résumé—On présente une solution d'équation unique pour obtenir la température et le flux thermique dans des corps finis monodimensionnels avec effet d'ailette. Quand cet effet est nul, cela conduit à un champ de température et de flux dans des corps finis monodimensionnels. Les corps peuvent être à une seule ou plusieurs couches. La solution correspond à la conduction variable mais elle donne la solution permanente pour les temps élevés. On utilise une solution récente de fonction de Green. La fonction de Green est calculée en utilisant la méthode intégrale Galerkin. Les exemples donnés ici sont pour des corps à une et deux couches.

EINE EIN-GLEICHUNGS-LÖSUNG FÜR WÄRMELEITUNG IN RIPPEN

Zusammenfassung—Es wird eine Ein-Gleichungs-Lösung zur Bestimmung von Temperatur und Wärmestrom in vielen endlichen eindimensionalen Körpern mit Rippeneffekten vorgestellt. Wenn der Rippeneffekt gleich Null gesetzt wird, ergibt sich die Temperatur und der Wärmestrom in endlichen eindimensionalen Körpern. Diese Körper können einfach oder mehrfach geschichtet sein. Die Ein-Gleichungs-Lösung gilt für den instationären Fall, liefert jedoch für lange Zeiten die stationäre Lösung. Es wird ein kürzlich entwickeltes, alternatives Lösungsverfahren verwendet, das auf der Green'schen Funktion beruht. Die Green'sche Funktion wird mit Hilfe einer auf dem Galerkin-Verfahren basierenden Integralmethode gelöst. Es werden Beispiele für einfach und zweifach geschichtete Körper vorgestellt.

РЕШЕНИЕ УРАВНЕНИЯ ТЕПЛОПРОВОДНОСТИ В РЕБРАХ, ВЫРАЖЕННОЕ ОДНИМ СООТНОШЕНИЕМ

Аннотация—Представлено выраженное одним соотношением решение уравнения для температуры и теплового потока в большом числе конечных одномерных тел с учетом эффекта оребрения. Температура и тепловой поток в конечных одномерных телах рассчитывались в предположении отсутствия эффекта оребрения. Исследуемые тела могут быть как однослойными, так и многослойными. Состоящее из одного соотношения решение уравнения получено для случаев нестационарной теплопроводности, однако при больших временах оно становится стационарным. Используется недавно разработанный альтернативный метод решения функции Грина, которая рассчитывалась интегральным методом Галеркина. Приведены примеры для одно- и двухслойных тел.